

Asymmetry with respect to the magnetic field direction in the interaction between the quantum states of two coupled superconducting rings

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 (Dated: February 2, 2008)

The interaction between the quantum states of two aluminum superconducting rings forming an 8-shape circular-asymmetric microstructure was examined under a threading magnetic flux and bias by an alternating current without a dc component. Quantum oscillations of the rectified dc voltage $V_{dc}(B)$ as a function of magnetic field were measured in the 8-shape microstructure at various bias ac currents and temperatures close to critical. Fourier and wavelet analyses of $V_{dc}(B)$ functions revealed the presence of various combination frequencies in addition to two ring fundamental frequencies, which suggests the interaction in the structure. Deviation of the $V_{dc}(B)$ function from oddness with respect to the magnetic field direction was found for the first time.

PACS numbers: 74.40.+k, 74.78.Na, 73.40.Ei, 03.67.Lx, 85.25.-j

The aim of the work is to study the expected interaction between two directly coupled asymmetric rings, using a recently discovered effect of ac voltage rectification in an asymmetric ring [1]. Time-averaged nonzero dc voltage $V_{dc}(B)$ was experimentally observed when a bias sinusoidal current (without a dc component) of an amplitude close critical and frequencies up to 1 MHz was transmitted through a single circularly asymmetric superconducting ring in a perpendicular magnetic field. The $V_{dc}(B)$ voltage as the function of B oscillates with the period $\Delta B = \Phi_0/S$, where Φ_0 is the superconducting magnetic flux quantum and S is the effective ring area [1]. Unlike $R(B)$ oscillations in the Little-Parks effect [2], the $V_{dc}(B)$ function is sign-alternating and odd with respect to the magnetic field direction [1]. Earlier, the behavior of superconducting loops in magnetic field was studied using $R(B)$ and $T_c(B)$ functions [2, 3].

The circular asymmetry of the structure under study (Fig. 1) makes it possible to use the $V_{dc}(B)$ function for the analysis of quantum behavior of the superconducting system. The ring quantum state is fully determined by the magnitude and direction of circulating current $I_p(B)$. Because the rectified voltage $V_{dc}(B)$ in a single asymmet-

ric ring is probably proportional to the ring circulating current $I_p(B)$ [1], measurements of the $V_{dc}(B)$ voltage can help determine the quantum state of the ring. Measuring $V_{dc}(B)$ in the system of different coupled asymmetric rings can be expected to provide information on the quantum state of each ring and the interaction between them.

The interest to the interaction in the system of superconducting asymmetric rings is due to that a similar ring with 10 nm thick walls at $T < 0.5T_c$ biased by a microwave current can potentially be used as an element for a novel superconducting flux qubit with quantum phase-slip centers (QPSC) [4]. Two directly coupled rings forming an 8-shape structure can be a prototype of two directly coupled flux qubits. In such a qubit, quantum phase-slip centers play the role of tunnel contacts, which are present in all known working superconducting flux qubits [5]. Such a qubit was theoretically considered in [6], but was not studied experimentally.

An aluminum structure $d = 72$ nm thick was fabricated by Al thermal deposition onto a silicon substrate, using lift-off process of electron-beam lithography. The structure is 8-shape and asymmetric structure with the wide, and narrow wires $w_w = 0.47$ and $w_n = 0.24$ μm wide, respectively (Fig. 1).

Average geometrical areas of the larger and the smaller rings are $S_L^g = 14.5$ μm^2 and $S_S^g = 8.5$ μm^2 . The parameters of the structure are the following: $R_{4.2} = 8.39$ Ω (normal-state resistance at $T = 4.2$ K), $R_{300}/R_{4.2} = 2.22$ (ratio of room to helium temperature resistances), $R_s = 0.33$ Ω (sheet resistance), $l = 25$ nm (an effective mean free path of electrons), $T_c = 1.324$ K (superconducting critical temperature), $\xi(0) = 170$ nm (superconducting coherence length at $T = 0$), and $\lambda_d(0) = 246$ nm (magnetic field penetration depth at $T = 0$).

In this work, we present an experimental observation of the deviation of the $V_{dc}(B)$ function from oddness in the 8-shape structure (Fig. 1). Figure 2 (bottom) shows

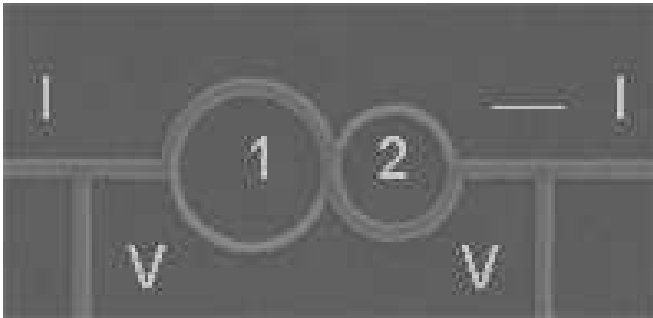


FIG. 1: Scanning electron micrograph of the 8-shape asymmetric structure with current and potential contacts. The longitudinal mark: 2 μm .

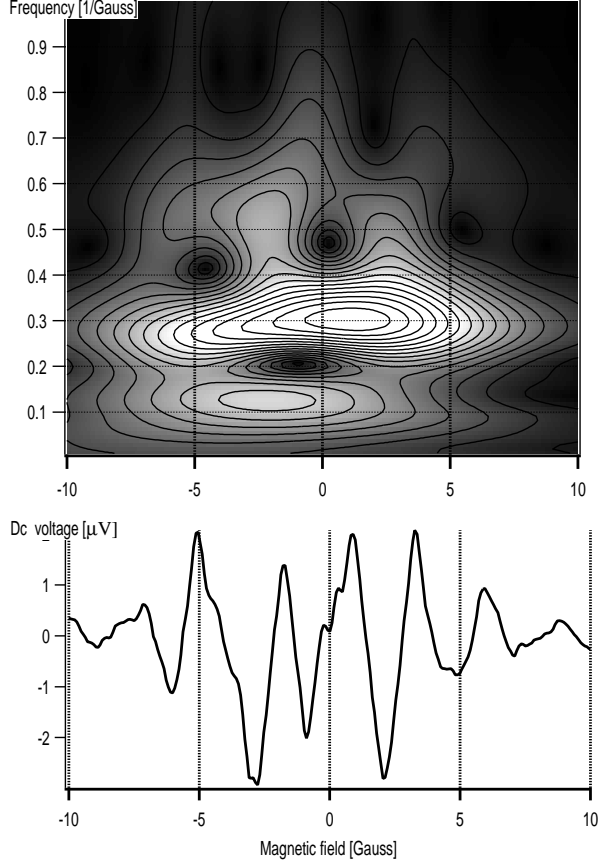


FIG. 2: Bottom panel: The rectified voltage $V_{dc}(B)$ in the structure at the bias sinusoidal current $\nu = 1.5$ kHz and amplitude $I_\nu = 2.8 \mu\text{A}$ at $T = 1.306$ K; the critical current in the zero magnetic field $I_c(T, B = 0) = 2.27 \mu\text{A}$ and the critical temperature $T_c = 1.324$ K. Top panel: The wavelet transform of the $V_{dc}(B)$ curve in the range -40 to +40 Gauss; the wavelet spectrum amplitude as the function of magnetic field (abscissa axis, Gauss) and frequency (ordinate axis, Gauss^{-1}) is presented along the z axis as different shades of gray (lighter areas correspond to larger amplitudes).

the rectified dc voltage $V_{dc}(B)$ in the structure biased by a sinusoidal current (without a dc component) of $\nu = 1.5$ kHz and amplitude I_ν close to critical. The deviation from the $V_{dc}(B)$ function oddness is clearly seen. The fast Fourier transform (FFT) of this not strictly odd function taken in the whole interval of magnetic fields gives an integral picture. In order to analyze the behavior of the $V_{dc}(B)$ function in negative and positive magnetic fields separately, we made a wavelet analysis of the curve in Fig. 2 (top).

The wavelet analysis [7] allows the decomposition of a signal into locally confined waves, wavelets. The result of the wavelet transformation of a $f(t)$ function is determined as $(Tf)(a, b) = |a|^{-\frac{1}{2}} \int dt f(t) \psi(\frac{t-b}{a})$, where $\psi(t)$ is the "mother" wavelet function. In our case, Morlet [7] "mother" function $\psi(t) = (\exp(i\gamma t) -$

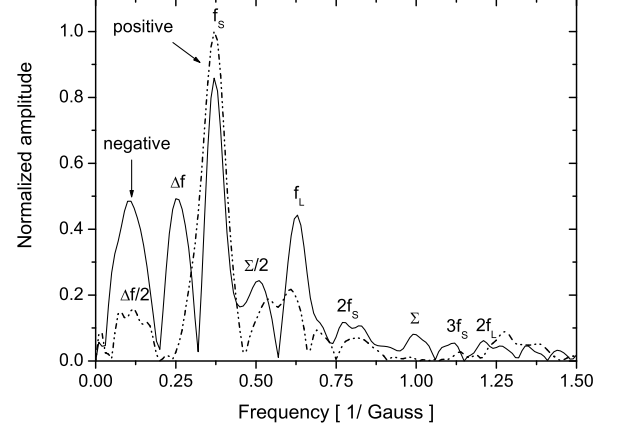


FIG. 3: The solid line presents the Fourier spectrum (FFT) from the left part of the $V_{dc}(B)$ curve taken in the range of negative magnetic fields (-40 to 0 Gauss) and extended to positive magnetic field in an odd way. The dash-dotted line presents the Fourier spectrum from the right part of the $V_{dc}(B)$ curve taken in the range of positive magnetic fields (0 to +40 Gauss) and extended to negative field in an odd way.

$\exp(-\frac{\gamma^2}{2}) \exp(-\frac{t^2}{2})$ was used. The parameter γ determines an optimum number of oscillations analyzed. The result of wavelet transformation is a function of two parameters. The value $(\frac{1}{a})$ corresponds to the frequency in the Fourier transformation. Each function $\psi(\frac{t-b}{a})$ localized around $t = b$ values. Fig. 2 (top) gives a three-dimensional view of the $V_{dc}(B)$ curve wavelet transform.

The amplitude of the wavelet transform is shown along the z axis as different shades of gray versus magnetic field (x axis) and frequency. i.e. the reciprocal of a certain period of oscillations (y axis). Lighter colors correspond to larger wavelet amplitudes. Closed curves are lines with similar amplitude values. The asymmetry of the wavelet amplitude with to magnetic field is evident.

Separate Fourier analysis of both parts of the $V_{dc}(B)$ function corresponding to negative and positive magnetic fields was also made. To improve the resolution of the FFT spectrum, the left (as well as the right) part of $V_{dc}(B)$ was extended to the corresponding region of positive (negative) fields in an odd way with respect to $B = 0$. The Fourier transforms of the resulting curves are shown in Fig. 3. We used 2^{12} uniformly distributed points in the -40 to +40 Gauss range. A set of amplitude peaks can be seen at certain frequencies, which are reciprocal to different oscillation periods in $V_{dc}(B)$. So, the fundamental frequencies f_S and f_L corresponding to the effective areas of the smaller and larger rings S_S and S_L , respectively, will be reciprocal to the corresponding oscillation periods for the smaller and larger rings, i.e. $f_S = 1/\Delta B_S = S_S/\Phi_0$ and $f_L = 1/\Delta B_L = S_L/\Phi_0$. The values of the fundamental frequencies expected from the

structure geometry are $f_S^g = 0.41 \text{ Gauss}^{-1}$, $f_L^g = 0.69 \text{ Gauss}^{-1}$. Indeed, the Fourier spectrum exhibits peaks in the regions of these values.

In addition to the fundamental frequencies f_S and f_L , the spectrum displays higher harmonics of the fundamental frequencies $f_{Sm} = mf_S$ and $f_{Lm} = mf_L$, difference and half-difference frequencies $f_\Delta = \Delta f = f_L - f_S$, $f_\Delta/2 = \Delta f/2$, and summation and half-summation frequencies $f_\Sigma = f_L + f_S$, $f_\Sigma/2$.

Low-frequency peaks determined by the oscillation attenuation field also observed. Moreover, spectral peaks are observed at frequencies, which are combinations of the above frequencies and higher harmonics of the fundamental frequencies. The presence of the combination frequencies suggests interactions in the structure.

The wavelet and FFT analyses (Figs. 2, 3) show that the response of different parts of the structure to the magnetic field and the interaction in the structure crucially depend on the magnetic field direction. For example, the difference and half-difference frequencies are mainly observed in the negative fields. When the amplitude of the bias ac current was much larger than the magnitude of the zero-field critical current, the magnetic asymmetry considerably decreased. These observations cannot be explained by the presence of extra uncontrolled direct or alternating currents and extra disregarded field. Magnetic asymmetry could have been explained by some sort of "freezing" of closed currents in the structure, but this is hardly the case because of the structure geometry. The understanding of this strange asymmetry requires further investigations.

In conclusion, the wavelet and FFT analyses of the $V_{dc}(B)$ function in the 8-shape structure have provided information on the quantum state of the system consisting of two rings and the interaction between the rings. An asymmetry in the interactions in the asymmetric superconducting structure has been observed, which depends on the external parameters and the structure geometry.

The authors are grateful to V. L. Gurtovoi, A. V. Nikulov, and V. A. Tulin for useful discussions. The work was financially supported in the framework of the program "Computations based on novel physical quantum algorithms", Information Technologies and Computer Systems Department of the Russian Academy of Sciences and the program "Quantum Macrophysics", Presidium of the Russian Academy of Sciences.

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